Teacher knowledge, curriculum materials, and quality of instruction: Unpacking a complex relationship

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The set of papers presented in this issue comprise a multiple-case study which attends to two instructional resources—teacher knowledge and curriculum materials—to understand how they individually and jointly contribute to instructional quality. We approach this inquiry by comparing lessons taught by teachers with differing mathematical knowledge for teaching who were using either the same or different editions of a US Standards-based curriculum. This introductory paper situates the work reported in the next four case-study papers by outlining the analytic framework guiding the exploration and detailing the methods for addressing the research questions.

Keywords: curriculum materials; mathematics; teacher knowledge; teaching quality

There is increasing consensus that, to improve student learning, greater attention needs to be paid to instruction itself (Hiebert and Grouws 2007, Raudenbush 2008). Current theories of instruction define the construct as interactions of teachers and students around content that occur in environments (Cohen et al. 2003, Hiebert and Grouws 2007). These interactions are shaped by how teachers and students deploy and mobilize their personal (e.g. knowledge, beliefs) and systemic (e.g. curriculum materials, school facilities, instructional time) resources. The past two decades have witnessed growing international interest in two key resources within the instructional system: teacher knowledge and curriculum materials. This interest is suggested by the publication of special issues on teacher knowledge (e.g. Ben-Peretz 2011, Tirosh 1999) and curriculum materials (e.g. Kulm and Li 2009); the issuing of volumes that focus squarely on the role of these resources within an international context (e.g. Sullivan and Wood 2008, Remillard et al. 2009); and the organization of special interest groups, seminars, and research forums in interna-
tional conferences (e.g. the Research Forum on *Teacher Knowledge and Teaching* in the 33rd conference of the International Group of the Psychology of Mathematics Education, Watson 2008). This interest seems to also cut across different subject matters, as implied by the inclusion of chapters or entire sessions that focus on either teacher knowledge or the curriculum in recent handbooks of various disciplines, ranging from mathematics education (e.g. Stein et al. 2007, Even and Tirosh 2008), to science education (Abell 2007, Atkin and Black 2007), to physical education (Penny 2006, Tsangaridou 2006).

Despite rapidly accumulating work examining the utility and utilization of each resource independently, research that foregrounds and explores both resources is scarce. For example, although research on teachers’ knowledge acknowledges the role curriculum materials play in shaping instruction, curriculum has largely been treated incidentally. Similarly, although teachers’ use of curriculum materials is often assumed to be influenced by their knowledge, studies exploring this association have typically not used objective measures of teacher knowledge. The importance of simultaneously attending to both resources has been both conceptually justified (e.g. Remillard 2005, Brown 2009) and empirically supported by the few studies situated at the intersection of teacher knowledge, curriculum use, and the quality of instruction (reviewed below). Attending to teachers’ knowledge and their use of curriculum materials could also yield useful recommendations for policy and teacher education, especially given recent studies that suggest that simply injecting ambitious curricula into the instructional system does not guarantee high-quality instruction (Tarr et al. 2008). By concurrently attending to these two resources, researchers could also respond to recent recommendations for exploring how teacher knowledge, *in conjunction with* other systemic resources and contextual factors, contribute to instructional quality (Kennedy 2010, Shechtman et al. 2010).

Thus, in this multiple-case study we simultaneously attend to both resources and explore how they separately and jointly contribute to instructional quality. Although our focus is on the teaching of mathematics—a convenience-sampling decision—mathematics instruction is used as a *case* to help us understand how teachers might marshal and employ their available resources and how this, in turn, might shape instructional quality. In particular, we focus on one type of teacher knowledge, namely teachers’ mathematical knowledge for teaching, and their use of a US Standards-based curriculum. By *mathematical knowledge for teaching* (MKT), we mean the ‘mathematical knowledge needed to perform the recurrent tasks of teaching mathematics to students’ (Ball et al. 2008: 399). By *curriculum materials* we refer to artifacts such as student textbooks, teacher guides, lesson plans, and instructional materials kits that, by communicating ideas and practices, can shape classroom activity (Ball and Cohen 1996: 8, Brown 2009: 21). We explore how these two resources contribute to the *mathematical quality of instruction*, which refers to a composite of features related to the mathematical—rather than the purely pedagogical—aspects of instruction (cf. Learning Mathematics for Teaching 2011, see more below).
Drawing on four comparative case studies of pairs or triads of teachers with differing MKT levels teaching the same or very similar lessons, we ask How do teachers’ MKT and curriculum materials separately and jointly contribute to the mathematical quality of instruction? More specifically, we pose the following questions:

(1) What is the unique contribution of teachers’ MKT to the mathematical quality of instruction?
(2) What is the unique contribution of curriculum materials to the mathematical quality of instruction?
(3) What is the joint contribution of teachers’ MKT and curriculum materials to the mathematical quality of instruction?
(4) What factors mediate the contribution of teachers’ MKT and curriculum materials to the mathematical quality of instruction?

In answering these questions with only a small sample of teachers, we cannot definitively address questions about causality; instead, following a case-method approach (Yin 2009) we develop tentative hypotheses about possible links based on our analysis of multiple cases.

The intention of this introductory paper is to provide background for the case studies reported in the next papers and to detail the methods pursued to answer our research questions. The paper begins by situating our work at the intersection of teacher knowledge, curriculum materials and their use by the teachers, and the quality of instruction. After presenting the framework that informed our research, we outline the characteristics of the curriculum considered in the case studies and describe the teacher participants, as well as the data sources and analyses.

Integrating research on teacher knowledge, curriculum materials, and instructional quality

In this section, we review studies linking instructional quality to either teacher knowledge or teachers’ use of curriculum materials. We focus, in particular, on mathematical knowledge for teaching (MKT), a construct advanced to capture the knowledge teachers need to effectively undertake the various tasks involved in teaching this subject, including knowing why an algorithm works, defining mathematical terms appropriately for students at a particular grade level, selecting and using representations and examples, analysing student errors, and evaluating student non-conventional ideas. MKT also includes ‘perspective, habits of mind, and appreciation that matters for effective teaching of the discipline’ (Ball et al. 2008: 399). Although originating from the US, over the last few years this construct has received increased attention in several countries, including Ireland (Delaney et al. 2008), Norway (Mosvold and Fauskanger 2009), Ghana (Cole 2009), Korea (Kwon 2009), and Indonesia (Ng 2011). We also focus on Standards-based curricula (cf. Senk and Thompson 2003), which, during the past two decades, have attracted scholarly interest in the US and beyond (cf. Lloyd et al. 2009). Because the strands of
research on teacher knowledge and teachers’ use of the available curriculum materials have largely developed in parallel, after reviewing studies on the contribution of each resource individually, we highlight the importance of working at their intersection. We conclude by outlining the analytic framework that informed our study.

**Teacher knowledge and the quality of instruction**

Driven largely by Shulman’s (1986) call for increased attention to teachers’ subject-matter knowledge, scholarly interest has focused on exploring the association between teacher knowledge and instructional quality. Initial work identified the potential affordances of strong knowledge and the possible limitations of weak knowledge on instructional quality (e.g. Cohen 1990, Stein et al. 1990, Borko et al. 1992, Putnam et al. 1992). In many of these exploratory studies, however, instructional quality and teacher knowledge were not measured independently. Subsequent research focused on three inter-related goals: to better conceptualize and understand the knowledge needed for the work of teaching mathematics; to develop and validate objective measures of teacher knowledge and instructional quality; and, by using these measures, to examine the effects of teacher knowledge on instructional quality more systematically. As a result, researchers worldwide have advanced different conceptualizations of teacher knowledge (e.g. Rowland et al. 2005, Davis and Simmt 2006, Ball et al. 2008, Center for Research in Mathematics and Science Teacher Development 2008); they have also developed different measures of teacher knowledge, ranging from multiple-choice or open-ended items administered on paper-and-pencil surveys (Hill et al. 2008, Baumert et al. 2010, Shechtman et al. 2010) to asking teachers to analyse videotaped clips (Kersting 2008). Furthermore, they have employed numerous ways to study instruction, including analysing the tasks in teacher-developed tests or samples of homework assignments (Baumert et al. 2010), observing and analysing instruction (Ding 2008, Hill et al. 2008, Izsák 2008), and studying teachers’ performance in simulated environments (Charalambous 2008).

Regardless of the diversity in conceptualizations, measurement tools, and methods, recent findings connecting teacher knowledge to the quality of instruction and consequently to student learning have been promising. For example, Baumert et al. (2010) found that, compared to teacher subject-matter knowledge, pedagogical content knowledge was a more powerful predictor of instructional quality as captured in the cognitive demand of the tasks used during instruction and the teacher–student interactions around these tasks. Similarly, in a recent study, Tchoshanov (2011) theorized three types of knowledge—knowledge of facts and procedures, knowledge of models and generalizations, and knowledge of concepts and connections; the latter was found to positively and significantly relate to instructional quality. In analysing 90 lessons taught by 10 teachers, Hill et al. (2008) provided evidence linking teachers’ MKT, the knowledge type considered in this multiple-case study, to the mathematical quality of
instruction: teachers with stronger MKT made fewer mathematical errors, responded more appropriately to students, and chose examples that helped students construct meaning of the targeted concepts and processes; teachers with weaker MKT were not successful at selecting and sequencing examples, presenting and elaborating upon textbook definitions, and using representations. MKT was also connected to delivering meaning-oriented instruction, as suggested by teachers’ performance in linking mathematical procedures to their underlying meaning through skillful use of representations and provision of mathematically appropriate explanations (Charalambous 2010).

Teacher knowledge has also been linked to student gains across theoretical conceptualizations and instruments. For example, in a large-scale longitudinal study of a representative sample of 10th-grade German students, Baumert et al. (2010) showed that teachers’ pedagogical content knowledge explained ~40% of the classroom variance in student achievement; net of other factors, two classes taught by teachers differing in their pedagogical content knowledge by two standard deviations differed by approximately half a standard deviation in their mean achievement by the end of the school year. Similarly, Hill et al. (2005) found that students taught by high-MKT elementary teachers experienced gains in their scores equivalent to ~2 weeks of instruction, compared to their counterparts taught by average-MKT teachers. Even recent studies that show MKT to contribute minimally to student learning gains (e.g. Shechtman et al. 2010) do not negate the potential contribution of teacher knowledge to student learning and call for a better conceptualization of the ways in which this knowledge, alongside other resources, contributes to instructional quality.

With only a few exceptions discussed below, most studies examining teacher knowledge kept the curriculum in the background—acknowledging its role, but not exploring its interaction with teacher knowledge in shaping instruction (e.g. Hill et al. 2008, Baumert et al. 2010). Studies that explored the role of the curriculum materials did so minimally (e.g. Rowland et al. 2005, Charalambous 2010), largely due to design limitations: because they sampled teachers using different curriculum materials, these studies could not support a systematic exploration of the interactions between teacher knowledge, curriculum materials, and instructional quality. Even a recent study that examined both the curriculum and MKT (Shechtman et al. 2010) focused only on teachers’ instructional decisions (e.g. choosing topics, setting goals) without investigating more proximal instructional activities, such as linking representations or building on students’ contributions.

Standards-based curriculum materials and the quality of instruction

Although the field of research on teachers’ use of curriculum materials emerged largely during the last quarter of the 20th century, it has grown tremendously over the last decade, partly due to major investments in curriculum material development made in response to the US National
Council of Teachers of Mathematics (NCTM) *Standards* (1989, 1991, cf. Lloyd *et al.* 2009). During the mid- and late 1990s, several projects in the US set out to develop curriculum materials that would realize the vision for teaching school mathematics outlined in the NCTM *Standards*. In contrast to the so-called ‘traditional’ or ‘conventional’ curricula, the *Standards*-based curricula that were published introduced new mathematical topics and pedagogical approaches; they included more problems set in realistic contexts and fewer exercises requiring only arithmetic or algebraic computation; they placed more emphasis on problem-solving, alternative solution methods, and discussions of multiple methods and answers; and they incorporated lessons designed to engage students in mathematical reasoning and communication and increase their problem-solving capacity and confidence in doing mathematics (cf. Senk and Thompson 2003). To fulfil these goals, teachers using these ambitious curricula were expected to build their instruction around big mathematical ideas, establish a mathematical discourse community that values conjecture and justification, guide students in sharing alternative solutions and explaining their approaches, and pose questions that elicit and challenge student thinking (Clarke 1997, Putnam 2003).

After the publication of these curricula, several studies were undertaken at all grade-levels¹ to examine whether *Standards*-based curricula ‘work’ (e.g. Huntley *et al.* 2000, Reys *et al.* 2003; see also Senk and Thompson 2003 for a collection of such studies). Indeed, these curricula were found to be at least as effective as more traditional curricula in promoting students’ procedural fluency, and more effective in enhancing students’ conceptual understanding, reasoning, and problem-solving. Yet, an evaluation of ~ 70 such comparative studies (National Research Council 2004) pointed to several design and methodological limitations, concluding that no robust argument can be made about the effectiveness of *Standards*-based curricula over commercially developed curricula.²

Recent studies appear to shift from examining whether these curricula work to exploring how they might work. Such studies (e.g. Arbaugh *et al.* 2006, Tarr *et al.* 2006, 2008) have documented that simply inserting *Standards*-based curricula into classrooms cannot on its own improve instructional quality, and that what matters is how these curricula are enacted. For example, Arbaugh *et al.* (2006) showed that even when using *Standards*-based curricula, some teachers still craft minimal opportunities for students to engage in high-level thinking and reasoning. Similarly, in a large-scale study involving more than 2500 students in 10 schools, Tarr *et al.* (2008) showed that students in classes using *Standards*-based curricula exhibit larger learning gains than their counterparts in classes using traditional curricula if they are engaged in activities such as making conjectures, justifying answers, and considering and linking multiple solution approaches—but, only 30% of the teachers using *Standards*-based curricula were found to engage their students in such activities. Collectively, these studies point to the prominent role that teachers have in implementing *Standards*-based curricula.

These studies have been complemented by more detailed examinations of teacher implementation of *Standards*-based curricula, aiming to
shed more light on why and how teachers influence curriculum effectiveness. Among the explanations offered are teachers’ fidelity of curriculum implementation (Manouchehri and Goodman 2000, Brown et al. 2009), orientations toward their curriculum (Remillard and Bryans 2004), beliefs (Remillard 1999, Collopy 2003), collaboration and reflection on practice (Clarke 1997, McDuffie and Mather 2009), and development of skills for successfully facilitating student work on rich mathematical tasks (Putnam 2003, Stein et al. 2008). An expanding body of studies (e.g. Remillard 2000, Schwartz 2006, Lloyd 2009, Stein and Kim 2009) has also explored whether curricula are educative for teachers—not only for students—and how this, in turn, might contribute to instructional quality.

Although teacher knowledge has been acknowledged as a potential contributor to the implementation of Standards-based curricula (cf. Stein et al. 2007, Sherin and Drake 2009), its role has been explored only minimally in studies on teachers’ use of these materials. In fact, despite hinting strongly at the role of teacher knowledge in enactment, most studies on teachers’ use of curriculum materials have not obtained objective measures of teacher knowledge, have not examined how teachers with different levels of knowledge use their available curriculum resources, and have not studied how both, in turn, inform instructional quality.

Toward integration: Working at the intersection of the two lines of research

This review suggests that research thus far has foregrounded either teacher knowledge or the curriculum materials as key contributors to instructional quality. Three reasons underscore the importance of attending to both simultaneously.

The first reason stems from theoretical frameworks designed to conceptualize the role of teachers and curriculum materials in instruction. According to these frameworks (Remillard 2005, Brown 2009), the teacher and the curriculum are engaged in a dynamic inter-relationship in which both are active participants and contributors to instructional outcomes. In this relationship, the teacher and the curriculum shape each other, and they both shape instruction. Hence, understanding how a teacher uses the available curriculum resources and how this, in turn, determines the quality of instruction requires an integrated analysis of teachers’ personal resources, the curriculum resources, and interactions between these resources. The NCTM Research Committee (2008) has also called for such an integration, pointing out that, because Standards-based curricula impose increased demands on teachers, more research is needed that links teacher knowledge, curriculum implementation, and student learning.

From a practical standpoint, working at the intersection of the two lines of research could yield important recommendations for policy and practice. Standards-based curricula impose increased demands on teachers, and it is important to understand how teachers respond to these demands, the ways in which their knowledge can support them in imple-
menting these curricula, and the degree to which teachers’ interactions with these curricula can enhance teacher knowledge. Similarly, exploring teacher knowledge in studies documenting discrepancies between curriculum visions and enactment could provide useful insights for curriculum and textbook development, teacher preparation, and teacher ongoing professional development.

The last warrant originates from three studies situated at the intersection of teacher knowledge and use of curriculum materials. In the first, Lloyd and Wilson (1998) examined how a high-school teacher’s strong and comprehensive content knowledge, measured through two in-depth interviews, supported him in implementing a reform-oriented curriculum unit on functions. The authors detailed how the teacher drew on both his knowledge and curriculum materials to structure a high-quality environment for his students, as revealed by his emphasis on meaning-making, as well as his capacity to build connections between representations and various concepts of functions. More recently, Ding (2008) has examined whether and how teachers’ knowledge contributes to handling student errors and difficulties. The six teachers in her study were observed teaching the same two lessons from a Standards-based curriculum; the curriculum included tasks lending themselves to addressing student errors and misconceptions on fractions. Teachers with stronger knowledge were found to be more sensitive to and more successful at handling student errors compared to teachers with weaker knowledge. Along similar lines, Izsák’s (2008) study of two sixth-grade teachers with different levels of knowledge using the same Standards-based lessons on fractions revealed notable differences in their use of representations and in how they interpreted and responded to students’ ideas.

Taken together, these studies suggest the importance of attending to the interactions of teacher knowledge and curriculum when exploring the quality of instruction. Yet these studies are limited in their reach; each examined only a few teacher cases, focused only on a single mathematical topic taught in a single grade, and attended only to limited aspects of instructional quality (e.g. using representations, providing explanations, or addressing student errors). Additionally, they did not engage in an analysis of the curriculum materials that would identify curriculum affordances and limitations to then examine how each interacts with teacher knowledge in shaping instruction. Hence, although providing important insights, these studies shed little light on a key issue—the interaction of teacher knowledge and their use of curriculum materials. Unpacking and better understanding this interaction is of critical importance to the field, and we explore this topic by varying the content and the grade-level taught.

To do so, we adopt a theoretical framework that derives from research on curriculum enactment. Following Remillard (2005), we view instructional quality as shaped by the interactions between teacher and curriculum resources (see Figure 1). For teacher resources, we mainly focus on teachers’ MKT, but drawing on Remillard’s (2005) and Brown’s (2009) frameworks, we also consider teachers’ beliefs about learning and teaching mathematics, orientations toward their curriculum materials, lesson goals, and teaching experience. For curriculum materials/resources, we focus on
representations of tasks and concepts. Representations of tasks ‘include instructions, procedures, and scripts [provided by the curriculum materials] that are intended for enactment by teachers and students’ (Brown 2009: 27). They also include recommendations for structuring a lesson, tasks for students, and plausible ways in which students might approach these tasks. In exploring this aspect of the curriculum, Brown also suggests considering other, less explicit, aspects of the curriculum materials, such as task sequencing. Representations of concepts refer ‘to the depiction and organization of domain concepts and their relationships through means such as diagrams, models, explanations, descriptions, and analogies’ (Brown 2009: 27). In line with Remillard’s (2005) framework, we also believe that specific contexts inform the teacher-curriculum relationship and, in turn, the quality of instruction.

As Figure 1 shows, we focus on one type of instructional quality: the mathematical quality of instruction, namely, a composite of five aspects (see Table 1): the richness of the mathematics in the lesson; teachers’ capacity to understand, follow, and weave student ideas into instruction; notational and linguistic precision and clarity in presenting the content; students’ contribution to meaning-making and reasoning; and the lesson coherence.

**Methods**

In this section, we present the analytic sample of the study (curriculum materials and teachers) and specify our data sources and analyses.
Table 1. Aspects of the mathematical quality of instruction.

I. Richness of the Mathematics: This aspect captures the depth of the mathematics offered to students. Rich mathematics focus either on the meaning of facts and procedures or on key mathematical practices. This aspect captures instructional elements such as:

- Linking and connections: Linking among different representations of mathematical ideas or procedures; drawing connections among different mathematical ideas, and drawing connections across representations and mathematical ideas.
- Explanations: Giving mathematical meaning to ideas, procedures, or solution methods.
- Multiple procedures or solution methods: Considering multiple solution methods or procedures for a single problem/task.
- Developing generalizations: Using specific examples to develop generalizations of mathematical facts or procedures.
- Mathematical language: Using dense and precise language fluently and consistently.

II. Working with Students and the Mathematics: This aspect captures whether teachers can understand and respond to students’ mathematically substantive productions (i.e. questions, claims, explanations, solution methods, ideas, etc. that contain substantial mathematical ideas) or mathematical errors. This aspect is intended to capture elements of instruction such as:

- Responding to student productions: The teacher responds to students in mathematically productive ways such as: identifying or highlighting salient mathematical points in student comments; building instruction on student ideas; clarifying a mathematical idea; pointing to similarities and differences.
- Remediating student difficulties and errors: Teacher remediates student difficulties and errors by focusing attention on meaning-making around problems, identifying the source of student errors or misconceptions, anticipating common student errors, and providing instruction that helps avoid error.

III. Errors and Imprecision: This aspect is intended to capture teacher errors or imprecision of language and notation, uncorrected student errors, or the lack of clarity/precision in the teacher’s presentation of the content. This aspect includes three problematic elements of instruction:

- Major mathematical errors or serious mathematical oversights (e.g. solving problems incorrectly, defining terms incorrectly, forgetting a key condition in a definition; equating two non-identical mathematical terms).
- Imprecision in language or notation: imprecision in use of mathematical symbols (notation), use of technical mathematical language, and use of general language when discussing mathematical ideas.
- Lack of clarity in teachers’ launching of tasks or presentation of the content.

IV. Student participation in meaning-making and reasoning: This aspect captures evidence of students’ involvement in cognitively activating classroom work and the extent to which students participate in and contribute to meaning-making and reasoning. Attention here focuses on student participation in activities such as:

- Providing explanations,
- Posing mathematically motivated questions or offering mathematical claims or counterclaims, and
- Engaging in reasoning and cognitively demanding activities (e.g. drawing connections among different representations, concepts, or solutions, explaining patterns, etc.)

V. Development of a coherent lesson trajectory: This aspect is intended to capture whole-lesson aspects of instruction, such as teachers’ selection and sequencing of tasks/examples, capacity to link different lesson activities, ability to move the mathematics along in a goal-directed manner (i.e., directionality-telos), and lesson cohesion.
Sampling decisions: Curriculum materials and teacher participants

The data used in this study are drawn from two projects exploring the mathematical quality of instruction conducted in districts in the southwestern US. The first project was conducted in a mid-sized racially diverse district containing 26 middle schools; 24 teachers from four of these schools were recruited for the project purposes (see more in Hill et al. 2011); eight of the teachers included in this study are drawn from this project. The second project recruited 10 teachers teaching in two districts serving families from a wide range of social, economic, and cultural backgrounds (see Hill et al. 2008 for more information); for this study, we focus on Rebecca, the only middle-school teacher sampled. An initial inspection of the data from both projects revealed that they included several instances in which middle-grade teachers with varying levels of MKT taught the same lessons (or lessons on very similar content) using either the same or different editions of the same curriculum. It is a close analysis of four such common lessons given by nine teachers that comprises this collection of papers (see Table 2).

All nine teachers in this study were using the Connected Mathematics Project (CMP) curriculum, one of the 13 Standards-based curriculum projects funded by the US National Science Foundation. This 3-year problem-centred curriculum was designed to promote high-level thinking and reasoning by engaging middle-grade students in activities such as drawing connections among different mathematical ideas and representations, sharing and comparing different solution approaches, making conjectures, developing generalizations, and attending to the underlying meaning of mathematical procedures (Ridgway et al. 2003, Lappan and Phillips 2009).

At each of Grades 6–8, the CMP curriculum includes eight units, each focusing on an important area of mathematics. Each unit is accompanied by a teacher’s guide and an assessment package designed to support the teacher in planning, teaching, assessing, and reflecting on student learning. Each teacher’s guide starts with a Mathematics Background essay intended to help teachers understand the goals, the content, and the forms of representation used in the unit. For this set of case studies, we considered lessons from four units: Grade-7 Comparing and Scaling, which deals with proportional reasoning (Case study 1), Grade-7 Accentuate the Negative, which focuses on operations with integers (Case study 2), Grade-8 Thinking with Mathematical Models, which introduces students to functions and modelling (Case study 3), and Grade-6 Bits and Pieces II, designed to help students develop algorithms for the four basic operations on fractions and decimals (Case study 4).

A CMP unit contains three to five investigations, each including two to five lessons inviting students to explore a key mathematical concept or process. Each CMP lesson consists of three phases. During the first phase, the ‘launch’, the teacher is expected to present the problem of the lesson and ensure that students are able to work productively on it. In ‘explore’, the second phase, students work on the problem individually or in groups;
Table 2. The four case studies and the teacher participants.

<table>
<thead>
<tr>
<th>Case study</th>
<th>Mathematical topic</th>
<th>Grade level</th>
<th>Pseudonym</th>
<th>MKT percentile</th>
<th>Credential type: Grades and (subject)</th>
<th>Grade ranges taught</th>
<th>Undergraduate mathematics</th>
<th>Years of experience</th>
<th>Curriculum edition used</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Comparing ratios</td>
<td>7</td>
<td>Mauricio</td>
<td>93</td>
<td>6–12 (mathematics and physical education)</td>
<td>5–9, 10–12</td>
<td>Major</td>
<td>16</td>
<td>CMP 2</td>
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<tr>
<td></td>
<td></td>
<td></td>
<td>Wanda</td>
<td>47</td>
<td>5–12 (mathematics)</td>
<td>5–9</td>
<td>Major</td>
<td>10</td>
<td>CMP 2</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Mercedes</td>
<td>42</td>
<td>K–9 (mathematics, social sciences, language arts)</td>
<td>5–9</td>
<td>—</td>
<td>2</td>
<td>CMP 2</td>
</tr>
<tr>
<td>2</td>
<td>Subtracting integers</td>
<td>7</td>
<td>Rebecca</td>
<td>22</td>
<td>Pre-credential</td>
<td>5–9</td>
<td>(unknown)</td>
<td>5</td>
<td>CMP 1</td>
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<tr>
<td></td>
<td></td>
<td></td>
<td>Bonita</td>
<td>7</td>
<td>K–9 (generalist)</td>
<td>K–4, 5–9</td>
<td>Graduate degree</td>
<td>11</td>
<td>CMP 2</td>
</tr>
<tr>
<td>3</td>
<td>Exploring linear</td>
<td>8</td>
<td>Monique</td>
<td>94</td>
<td>K–9 (generalist)</td>
<td>K–4, 5–9</td>
<td>Graduate degree</td>
<td>25</td>
<td>CMP 2 (supplemented with CMP 1)</td>
</tr>
<tr>
<td></td>
<td>relationships</td>
<td></td>
<td>Brad</td>
<td>31</td>
<td>K–9 (generalist)</td>
<td>5–9</td>
<td>—</td>
<td>15</td>
<td>CMP 1</td>
</tr>
<tr>
<td>4</td>
<td>Adding and subtracting</td>
<td>6</td>
<td>Marie</td>
<td>89</td>
<td>K–9 (generalist) and 10–12 (reading)</td>
<td>K–4, 5–9</td>
<td>—</td>
<td>37</td>
<td>CMP 2</td>
</tr>
<tr>
<td></td>
<td>fractions</td>
<td></td>
<td>Waleska</td>
<td>50</td>
<td>K–9 (generalist)</td>
<td>5–9</td>
<td>Minor</td>
<td>12</td>
<td>CMP 2</td>
</tr>
</tbody>
</table>
during that time, the teacher circulates in the classroom, monitors student work, and provides support when needed. The teacher also takes stock of important student strategies and solutions that can be shared and discussed in the next phase. During this last phase, ‘summarizing’, the teacher initiates and facilitates a whole-class discussion during which strategies used to solve the problem are shared and compared, generalizations are drawn, mathematical processes and concepts are abstracted, and connections among different mathematical ideas are established (Lappan and Phillips 2009). To support teachers’ work in each lesson, CMP lists examples of questions for eliciting or pushing on student thinking.

The first CMP edition, widely known as CMP 1, was released in 1998 after a 5-year period of development, piloting, field-testing, and revising (Ridgway et al. 2003). Based on feedback received from teachers around the US, a new round of field-testing and revising was undertaken, which resulted in a new CMP edition (commonly known as CMP 2) released in 2006. As shown in Table 2, two teachers from our sample were using CMP 1, six teachers were using CMP 2, and one teacher supplemented CMP 2 with additional CMP 1 problems. Table 2 also suggests that the teacher participants differed in the credentials earned, undergraduate mathematics studies, grade ranges taught, and years of experience. Most of the teachers had a Kindergarten through Grade-9 (K-9) credential; three teachers also had either a major or a minor in mathematics. The teachers had taught grades K-9 (three teachers), 5–9 (five teachers), and 5–12 (one teacher); at the time of data collection, the teacher with the least experience had only taught mathematics for 2 years, while the most experienced teacher was in her 37th year of teaching ($\bar{x} = 14.77$, SD = 10.02). The teachers also differed in their MKT, as discussed below.

### Data sources

The work reported here is based on analyses of a rich corpus of data including videotaped lessons, post-lesson interviews, curriculum materials, general interviews, and paper-and-pencil surveys measuring teachers’ MKT.

#### Videotaped lessons

Eight of the teacher participants had six of their lessons videotaped in two waves; Rebecca, the teacher drawn from the second project, had nine of her lessons videotaped in three waves. In all cases, the lessons were videotaped by videographers who were given instructions on how to capture general classroom events, teacher and student audio, and teacher and student productions shared during the lesson. The videotapes were then transcribed, with an estimate of 95% of teacher and 70% of student utterances captured audibly. In the papers that follow, we largely focus on one lesson per teacher, but we occasionally expand our focus on all lessons to determine whether the instructional features observed in the focal lesson were consistently present across the teacher’s lessons.
Post-lesson interviews and curriculum materials

After completing a videotaped lesson, the teachers responded to a set of questions concerning their lesson goals, their planning and preparation for the lesson, and any mathematical struggles they encountered in preparing or teaching the lesson. The teachers were also asked to identify any aspects of the lesson they found surprising, discuss what their students appeared to have learned or still struggled with, and describe any modifications they would introduce if they had to teach the lesson again. These post-lesson interviews were videotaped and transcribed. Because teachers listed the materials and lesson taught, we had access to the pages from the teacher’s guide and the student textbook that corresponded to the lessons under consideration.

General interviews

Teachers participated in two hour-long interviews that captured their orientation toward and use of curriculum materials, their beliefs about teaching and learning mathematics, and their typical teaching practices. The interviews also included a ‘clinical interview’ portion during which the teachers were asked to solve and discuss MKT items (e.g. evaluating a non-conventional student contribution, discussing the underlying meaning of a mathematical procedure). We drew on the general interviews to distil information on teachers’ orientations toward their curriculum materials and their overall views about mathematics and its teaching and learning. We used the ‘clinical interview’ portions to glean information on teachers’ ‘local’ MKT—namely, their MKT with respect to the specific mathematical content they were observed teaching in the lessons under investigation. For example, if the teacher participants taught ratios, we explored their performance and thinking on items directly related to this topic.

Paper-and-pencil MKT surveys

The teachers completed a pair of multiple-choice MKT surveys that included items on numbers and operations, algebra, and proportional reasoning. The items captured two elements of teachers’ MKT: common content knowledge and specialized content knowledge (for more information on the development of these surveys and for sample items, see Hill et al. 2008). Teachers’ responses to the survey items were used in two ways. First, they provided information on teachers’ ‘global’ MKT. This was done by entering these responses to an Item-Response-Theory model and scoring them alongside data obtained from a US nationally representative sample of middle-school teachers (see Hill 2007). As Table 2 shows, the study participants covered a wide range of ‘global’ MKT levels, with their scores extending from the 7th to the 94th percentile of the US sample; according to their percentile rankings, the sampled teachers can be clustered into three groups corresponding to high-MKT (Moni-
que, Mauricio, and Marie), mid-MKT (Waleska, Wanda, Mercedes, and Brad), and low-MKT levels (Rebecca and Bonita). Second, the teachers’ responses to items that matched the lessons described in the four case studies provided additional information on teachers’ ‘local’ MKT, especially when such information was not available from the ‘clinical interview’ portions of the general interviews.

**Data analyses**

Our data analysis was primarily qualitative and mainly consisted of a mixture of both hypothesis testing and generation. For the relatively more established notions on how MKT and the curriculum materials alone contribute to instructional quality (Research Questions 1 and 2), we followed Yin’s (2009) recommendation of testing theoretical propositions against the data from different cases. To this end, we started with certain hypotheses emanating from prior studies exploring how these two resources inform instruction. Using a pattern-matching strategy (Yin 2009: 137–138), we tested the predicted contribution of each resource to the mathematical quality of instruction against the data from each case and, if needed, refined the assumed patterns of contribution to fit the study data. In this approach, the teacher pair or triad within each case study was used as a *theoretical* replication (Yin 2009: 54): because the teachers varied in their MKT levels and/or in the curriculum edition they were using, the quality of their instruction was expected to vary, but for anticipated reasons.

In pursuing this analysis, we were also open to what Yin (2009: 133–135) calls ‘rival explanations’. In particular, for the more novel examination of how MKT and the curriculum interact to inform instructional quality, and to also investigate other potential contributors to instructional quality (Research Questions 3 and 4), we explored two types of rival explanations: a ‘commingled rival explanation’ (Yin 2009: 135), according to which the quality of instruction might not be explained by the individual contribution of MKT and curriculum alone, but by their joint contribution; and a ‘direct rival explanation’ (Yin 2009: 135), according to which other factors within the instructional milieu might contribute to the observed quality of instruction.

Because we did not have solid hypotheses about the potential interactive contribution of MKT and the curriculum or the contribution of other factors to instructional quality, to answer the latter two questions we followed a grounded-theory approach (Strauss and Corbin 1998), generating hypotheses from each case study, and then testing whether these hypotheses held across the four case studies of this collection. We also pursued this approach when we did not have specific hypotheses guiding our exploration of the unique contribution of MKT and the curriculum to the aspects of instruction examined in our study (see Table 1). We detail our overall analytic approach below.

Our analysis proceeded through six stages. In the first stage, we watched the videotaped lessons and wrote detailed analytic memos
narrating the unfolding of the lessons. The purpose of writing these memos was to analyse each lesson in light of the five aspects of the mathematical quality of instruction. We did not code for the specific elements identified within each aspect (see Table 1), but used these elements and the associated overarching framework as both a viewing lens and an organizer for our analysis. The detailed analytic memos that resulted at the culmination of the first stage provided the bedrock for the analysis undertaken in the next stages: they were used to confirm or amend our hypotheses or to generate and test new hypotheses.

In the second stage, we drew on our analytic memos and on information on teachers’ MKT to explore the potential contribution of MKT to instructional quality. To this end, building on the conceptualization of MKT (Ball et al. 2005, 2008) and on prior studies exploring the contribution of MKT to instructional quality (Hill et al. 2008, Charalambous 2010), we formulated the following hypothesis:

Hypothesis 1: Teachers with stronger MKT (‘global’ and/or ‘local’) are expected to (a) deliver lessons that support meaning-making (through their skilful use of representations, provision of explanations, use of dense mathematical language, and their work around developing generalizations or linking multiple procedures); (b) understand and appropriately respond to students’ mathematical contributions and difficulties; (c) avoid errors and imprecision in language and notation while presenting the content; and (d) offer coherent lessons that unfold in ways that support meaning-making and understanding. In contrast, teachers with lower MKT levels are expected to offer instruction of lower quality in the foregoing instructional aspects.

Because previous studies have yielded mixed results on the extent to which teachers with high levels of MKT can immerse their students in cognitively demanding activities and maintain the level of cognitive activation and reasoning (e.g. Hill et al. 2008, Mitchell 2009, Shechtman et al. 2010), we did not have any pre-set hypothesis linking MKT and the quality of instruction for this particular instructional aspect, and we were open to hypotheses emerging from our data.

The third stage of our analysis pertained to examining the contribution of the curriculum materials to instructional quality. To this end, we took into consideration the characteristics and goals of Standards-based curricula in general, and the CMP curriculum in particular, as summarized above. We also considered previous research on how educative materials can support lesson enactment (e.g. Ball and Cohen 1996, Clarke 1997, Putnam 2003, Davis and Krajcik 2005, Schwartz 2006). Using these pieces of information, we formulated the following hypothesis, which we then tested against data from our analytic memos:

Hypothesis 2: If providing adequate levels of support, the CMP materials can enable teachers to offer high-quality lessons by helping them (a) provide mathematically rich and meaning-oriented instruction; (b) avoid mathematical errors and notational/linguistic imprecision; and (c) engage students in cognitively demanding activities that expect them to make meaning and reason mathematically. Provided that these materials also
present information on student misconceptions and difficulties, they can also scaffold teachers to effectively respond to student productions. In contrast, when not providing adequate levels of support, CMP materials can lead to lower-quality instruction in the aforementioned areas.

We did not have any initial hypotheses about the potential role of the curriculum materials in developing a coherent lesson trajectory (fifth instructional aspect), largely due to the increased demands that Standards-based curricula impose on teachers (cf. Stein et al. 2007) and also because these curricula, unlike traditional curricula, do not necessarily unfold from easier to more complex activities. Hence, in exploring this potential association, we were open to hypotheses emerging from our data.

To test the second hypothesis, we first needed to determine the level of support the curriculum materials provided for lesson enactment. To this end, we investigated the presentation and treatment of the content in the curriculum materials, as well as the scaffolds provided to teachers to understand and teach the content. In particular, we analysed the teacher's guides, focusing first on the Mathematics Background essays; we examined whether the essays sufficiently clarified the key mathematical ideas of the lesson and adequately explained how models/representations proposed to scaffold student learning could be used. We then examined the presentation of the content in the Investigation where each lesson was situated; here, we considered the goals outlined for the investigation as a whole, the goals suggested for the focal lesson, and the sequencing and support of key mathematical ideas and practices being developed in the investigation. Next, we scrutinized the pages pertaining to the focal lesson, attending, in particular, to any clarifications of key mathematical ideas, suggestions provided for launching the problem(s), questions listed for scaffolding students' work, any scripts outlined for modelling how representations could be used, and the organization of pertinent information for lesson enactment. We also examined the student textbooks to explore the ordering of the tasks that students were expected to complete and for additional scaffolds for understanding and teaching the lesson. Finally, we analysed the post-lesson interviews to get insights into whether the teachers themselves found the curriculum materials helpful in their lesson preparation and teaching. For the case studies in which teachers were using only one CMP edition (see Table 2), we explored the extent to which the aspects of the curriculum materials outlined above were sufficiently supportive to enhance instructional quality. If the teachers were using different CMP editions, we compared the level of support for lesson enactment provided by each edition; when differences were identified, we explored whether the more supportive edition resulted in higher-quality instruction.

The fourth and fifth stages concerned generating alternative explanations, especially when our data did not support our original propositions on the contribution of MKT and the curriculum. In the fourth stage, we searched for alternative explanations by looking at the joint contribution of MKT and the curriculum to instructional quality. For example, in this stage, we considered exploring whether teachers' strong MKT could compensate for limitations in the curriculum materials, or, conversely, limita-
tions in teachers’ MKT could be compensated by the curriculum presentation and treatment of the content. If this compensation held, we were curious to figure out if and how it informed instruction, and particularly to which instructional aspects outlined in Table 1 it contributed the most. In the fifth stage, and when these two key resources, separately or jointly, failed to explain instructional quality, we contemplated other explanations. We did so by scrutinizing the general and post-lesson interviews, with an eye toward identifying factors that could influence the quality of instruction, such as teachers’ beliefs, lesson goals, and orientations toward their curriculum.

The four ‘individual case reports’ (cf. Yin 2009: 56–57) that comprise the next four papers capture the product of our work in the preceding five stages; these papers provided the data for the cross-case analysis undertaken in the last stage. In this cross-case analysis, each case study served as a replication (Yin 2009: 54), which helped explore the tenability of our hypotheses—both those tested and those generated—across different mathematical topics and grade-levels taught. Additionally, this cross-case analysis enabled examining the generality of the findings emerging from the four case studies while at the same time detailing the conditions under which these findings occurred (cf. Borman et al. 2006, Yin 2009). To organize our work in this stage, we created a table displaying the data from the individual case studies and looked across these studies for common themes. The results of this last stage are presented in the Conclusions paper, which comprises the cross-case report.

The four case studies of this collection

The pairs or triads of teachers featured in the first three case studies differed in their knowledge level and/or in the support their curriculum materials provided for lesson enactment. This allowed examining the main and joint contribution of MKT and the curriculum to instructional quality in the context of teaching a rich mathematics problem that admits several solution approaches (Case study 1), teaching integer operations for conceptual understanding and procedural fluency (Case study 2), and teaching a linear algebra lesson (Case study 3). With notable differences in their dispositions toward their curriculum, the cases portrayed in the fourth case study supported exploring the potential influence of this factor on instruction, and, more generally, helped investigate whether the combination of MKT and the curriculum materials alone sufficiently explains the mathematical quality of instruction.

We opted for a common structure across the four case studies. After a brief introduction, each paper presents and analyses the curriculum materials considered in the focal lesson. We then present background information about the teachers featured in the paper, including their ‘global’ and ‘local’ MKT, the school context in which they were working, their experience, their orientations toward their curriculum materials, and their beliefs about teaching and learning mathematics. Following that, we present the teachers’ lesson enactments. Each case study concludes with a dis-
cussion, in which we address the first three research questions, and, if applicable, the fourth research question.

We would like to conclude this introductory paper with two closely related caveats. First, although one might read into our papers a critique of CMP, this is certainly not what we intended. The case studies that follow show that, in some cases, CMP supported teachers in providing better instruction compared to what might have occurred otherwise. Also, because CMP is an ambitious curriculum, often imposing greater demands on teachers than other curriculum materials, it is not surprising that some of the teacher participants experienced challenges in their attempts to implement the curriculum. One also needs to keep in mind that we focused only on a restricted sample of CMP units, lessons, and tasks; hence, our findings are by no means representative of the quality of either of the two CMP editions. Additionally, while analysing the CMP materials, we were mindful of the challenges inherent in developing educative curricula without producing voluminous materials which would be hard to use (cf. Lappan and Phillips 2009).

Second, we are cognizant that teaching is a complex and dynamic process which imposes many demands on teachers and confronts them with several dilemmas (Lampert 1985, Ball 1993, Leinhardt 1993). This is especially true when it comes to using ambitious curriculum materials (Clarke 1997), which tax teachers’ knowledge and skills in ways that typical materials do not, and the support for such knowledge and skills often varies. Hence, our depiction of the cases that follow should be read more as an analysis of the work of teaching rather than as a critique of the teachers featured in this set of papers. In fact, we are grateful to these teachers who graciously opened their classroom doors and contributed their lessons for scrutiny and learning from actual practice. In short, we invite the reader to consider this collection of papers as another study aiming to help us understand the conditions under which teachers’ use of ambitious curricula, such as CMP, can help realize the formidable task for which these curricula have been designed: to improve the quality of instruction and consequently student learning.

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Notes

1. The US school system comprises three grade-levels: the elementary-school level (Grades 1–5, ages 6–11), the middle-school level (Grades 6–8, ages 12–14), and the high-school level (Grades 9–12, ages 15–18).

2. In fact, a recent study employing a more elaborated design (Agodini & Harris 2010) produced mixed results, showing one Standards-based and one commercially-devel-
oped curriculum to be more effective in improving first-graders’ achievement compared to another pair of Standards-based and a conventional curriculum.

3. All the teacher names used in this paper are pseudonyms.

References


